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Algebraic expressions for $SU(3) \supset R(3)$ Wigner coefficients for $(\lambda\ 0) \times (3\ 0)$

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Abstract. In this short paper an algebraic expression for $(\lambda\ 0) \times (3\ 0)$ reduction Wigner coefficients in the $SU(3) \supset R(3)$ physical basis is obtained using a building-up process. The $SU(3)$ U-functions are also given.

Since the early work on $SU(3)$ in the shell model [1, 2], the $SU(3)$ group has been widely used in nuclear and particle physics [2–9]. Many authors studied the $SU(3)$ group in the Gelfand basis [10–12]. With the development of the interacting boson model for collective motion in even–even nuclei in nuclear physics [13], $SU(3)$ has been widely used in the description of rotational motion in deformed nuclei.

In order to obtain the wavefunction, one needs to know the Wigner coefficients in the $SU(3) \supset R(3)$ basis. Extensive tables of algebraic expressions have been given by Vergados [14]. These tables are sufficient for the sd-shell model calculations and for the interacting boson model calculations involving only the s and d bosons. A computer program is also available to calculate these coefficients numerically [15, 16]. With the advent of vector coherent states (VCS) methods [17], the $SU(3) \supset R(3)$ Wigner coefficients are constructed in VCS for $(\lambda\mu) \times (2\ 0)$, with λ and μ arbitrary [18].

In the new development of the interacting boson model, the pf bosons are introduced in order to describe the negative parity collective states as well [19]. One of the advantages of these algebraic models is the analytic formula for many quantities, e.g. the electric and magnetic transition rates. In order to gain an algebraic expression for the electric and magnetic transition rates, one needs to know the algebraic expression for $(\lambda\ 0) \times (3\ 0)$ Wigner coefficients. For instance, the lowest negative parity collective band in the rotational limit involves the $(\lambda+3\ 0)$ representation. It is the purpose of this short paper to generalize the work of Vergados for these cases.

The building-up method of Vergados [14] has been applied. Specifically, equation (14) of [14] is used in calculating the Wigner coefficients:

$$
\begin{aligned}
&\langle(\lambda_1\mu_1)\kappa_1 L_1; (\lambda_{23}\mu_{23})\kappa_{23}L_{23}\|(\lambda\mu)\kappa L\rangle \\
&\qquad \times U((\lambda_1\mu_1)(\lambda_2\mu_2)(\lambda\mu)(\lambda_3\mu_3); (\lambda_{12}\mu_{12})(\lambda_{23}\mu_{23})) \\
&\quad = \sum_{L_{12}l_2l_3} \langle(\lambda_1\mu_1)\kappa_1 L_1; (\lambda_2\mu_2)\kappa_2 L_2\|(\lambda_{12}\mu_{12})\kappa_{12}L_{12}\rangle \\
&\qquad \times \langle(\lambda_2\mu_2)\kappa_2 L_2; (\lambda_3\mu_3)\kappa_3 L_3\|(\lambda_{23}\mu_{23}\kappa_{23}L_{23}\rangle \\
&\qquad \times \langle(\lambda_{12}\mu_{12})\kappa_{12}L_{12}; (\lambda_3\mu_3)\kappa_3 L_3\|(\lambda\mu)\kappa L\rangle U(L_1L_2LL_3; L_{12}L_{23})\,. \qquad (1)
\end{aligned}
$$

Table 1. The $SU(3)$ U-function $U((\lambda_1 0)(2\ 0)(\lambda\ \mu)(1\ 0); (\lambda_{12}\ \mu_{12})(3\ 0))$. A – sign means the value does not exist.

$(\lambda_{12}\mu_{12})$	$(\lambda\mu) = (\lambda_1 + 2\ 0)$	$(\lambda\mu) = (\lambda_1\ 1)$	$(\lambda\mu) = (\lambda_1 - 2\ 2)$
$(\lambda_1 + 3\ 0)$	1	–	–
$(\lambda + 1\ 1)$	$\sqrt{\frac{\lambda_1}{3(\lambda_1+2)}}$	$\sqrt{\frac{2(\lambda_1+3)}{3(\lambda_1+2)}}$	–
$(\lambda_1 - 1\ 2)$	–	$\sqrt{\frac{2(\lambda_1-1)}{3\lambda_1}}$	$\sqrt{\frac{\lambda_1+2}{3\lambda_1}}$
$(\lambda_1 - 3\ 3)$	–	–	1

Table 2. $\langle(\lambda\ 0)L_1;\ (3\ 0)l\|(\lambda + 3\ 0)L\rangle$.

	L_1	$\lambda - L =$ odd
$l = 1$	$L - 1$	$\sqrt{\frac{3(\lambda-L+3)(\lambda+L+2)(\lambda+L+4)L}{5(\lambda+1)(\lambda+2)(\lambda+3)(2L+1)}}$
	$L + 1$	$-\sqrt{\frac{3(\lambda-L+1)(\lambda-L+3)(\lambda+L+4)(L+1)}{5(\lambda+1)(\lambda+2)(\lambda+3)(2L+1)}}$
$l = 3$	$L - 1$	$-\sqrt{\frac{3(\lambda-L+3)(\lambda+L+2)(\lambda+L+4)(L-1)L(L+1)}{5(\lambda+1)(\lambda+2)(\lambda+3)(2L-3)(2L+1)(2L+3)}}$
	$L + 1$	$\sqrt{\frac{3(\lambda-L+1)(\lambda-L+3)(\lambda+L+4)L(L+1)(L+2)}{5(\lambda+1)(\lambda+2)(\lambda+3)(2L-1)(2L+1)(2L+5)}}$
	$L - 3$	$\sqrt{\frac{(\lambda+L)(\lambda+L+2)(\lambda+L+4)(L-2)(L-1)L}{(\lambda+1)(\lambda+2)(\lambda+3)(2L-3)(2L-1)(2L+1)}}$
	$L + 3$	$-\sqrt{\frac{(\lambda-L-1)(\lambda-L+1)(\lambda-L+3)(L+3)(L+2)(L+1)}{(\lambda+1)(\lambda+2)(\lambda+3)(2L+1)(2L+3)(2L+5)}}$

Table 3. $\langle(\lambda\ 0)L_1;\ (3\ 0)l\|(\lambda + 1\ 1)L\rangle$.

	L_1	$\lambda - L =$ odd
$l = 1$	$L - 1$	$-\sqrt{\frac{(\lambda+L+2)(L+1)}{5\lambda(\lambda+1)(\lambda+3)(2L+1)}}(\lambda + 3 - 3L)$
	$L + 1$	$-\sqrt{\frac{(\lambda-L+1)L}{5\lambda(\lambda+1)(\lambda+3)(2L+1)}}(\lambda + 6 + 3L)$
$l = 3$	$L - 1$	$\sqrt{\frac{(\lambda+L+2)(L-1)}{5\lambda(\lambda+1)(\lambda+3)(2L-3)(2L+1)(2L+3)}}(\lambda(L + 6) - 3(L^2 - 6))$
	$L + 1$	$-\sqrt{\frac{(\lambda-L+1)(L+2)}{5\lambda(\lambda+1)(\lambda+3)(2L+5)(2L+1)(2L-1)}}(\lambda(-L + 5) - 3(L^2 + 2L - 5))$
	$L - 3$	$-\sqrt{\frac{3(\lambda+L)(\lambda-L+3)(\lambda+L+2)(L-2)(L-1)(L+1)}{\lambda(\lambda+1)(\lambda+3)(2L-3)(2L-1)(2L+1)}}$
	$L + 3$	$-\sqrt{\frac{3(\lambda-L-1)(\lambda+L+4)(\lambda-L+1)(L+3)(L+2)L}{\lambda(\lambda+1)(\lambda+3)(2L+1)(2L+3)(2L+5)}}$
	L_1	$\lambda - L =$ even
$l = 1$	L	$\sqrt{\frac{(\lambda-L+2)(\lambda+L+3)}{5\lambda(\lambda+1)}}$
$l = 3$	$L - 2$	$\sqrt{\frac{(\lambda+L+1)(\lambda+L+3)(L-2)(L-1)}{\lambda(\lambda+1)(2L+1)(2L-1)}}$
	$L + 2$	$\sqrt{\frac{(\lambda-L)(\lambda-L+2)(L+2)(L+3)}{\lambda(\lambda+1)(2L+1)(2L+3)}}$
	L	$-\sqrt{\frac{6(\lambda-L+2)(\lambda+L+3)(L-1)(L+2)}{5\lambda(\lambda+1)(2L+3)(2L-1)}}$

Here we have dropped the floating index from the original equation in [14] because in our case it is multiplicity-free. We have chosen $(\lambda_1\mu_1) = (\lambda\ 0)$, $(\lambda_2\mu_2) = (2\ 0)$ and

$(\lambda_3\mu_3) = (1\ 0)$. $(\lambda_{23}\ \mu_{23}) = (3\ 0)$. $(\lambda_{12}\mu_{12})$ maybe different for different $(\lambda\mu)$'s. It is as follows: (i) for $(\lambda\mu) = (\lambda_1 + 3\ 0)$, $(\lambda_{12}\mu_{12}) = (\lambda_1 + 2\ 0)$; (ii) for $(\lambda\mu) = (\lambda_1 + 1\ 1)$, $(\lambda_{12}\mu_{12}) = (\lambda_1 + 2\ 0)$ or $(\lambda_1\ 1)$; (iii) for $(\lambda\mu) = (\lambda_1 - 1\ 2)$, $(\lambda_{12}\mu_{12}) = (\lambda_1\ 1)$ or $(\lambda_1 - 2\ 2)$; (iv) for $(\lambda\mu) = (\lambda_1 - 3\ 3)$, $(\lambda_{12}\mu_{12}) = (\lambda_1 - 2\ 2)$.

A minor difference from [14] in using the building-up equation (1) is that it is used to obtain the $SU(3) \supset R(3)$ Wigner coefficients and the $SU(3)$ U-function simultaneously. This is done in two steps. First, for a given $(\lambda\mu)$ representation, all the unnormalized Wigner coefficients defined as $\langle(\lambda_1\mu_1)\kappa_1 L_1; (\lambda_2\mu_2)\kappa_2 L_2\|(\lambda_3\mu_3)\kappa_3\mu_3\rangle \times U((\lambda_1\mu_1)(\lambda_2\mu_2)(\lambda\mu)(\lambda_3\mu_3); (\lambda_{12}\mu_{12})(\lambda_{23}\mu_{23}))$ are calculated for all possible $\kappa_1 L_1$ and $\kappa_2 L_2$. Secondly, using the normalization property of the Wigner coefficients,

$$\sum_{\kappa_1 L_1 \kappa_2 L_2} \langle(\lambda_1\mu_1)\kappa_1 L_1; (\lambda_2\mu_2)\kappa_2 L_2\|(\lambda\mu)\kappa L\rangle^2 = 1 \tag{2}$$

the sum of all the unnormalized Wigner coefficients squared for a given $(\lambda\mu)$ should give the square of the $SU(3)$ U-function. This also provides a rigorous check on the Wigner coefficients. With the choice of phase convention in the Vergados basis, the $SU(3)$ U-function is a real number, and its sign can be determined. The $SU(3)$ U-function obtained in the present calculation is given in table 1.

The Wigner coefficients for $(\lambda\mu) = (\lambda_1 + 3\ 0)$ are given in table 2, for $(\lambda\mu) = (\lambda_1 + 1\ 1)$ in table 3, for $(\lambda\mu) = (\lambda_1 - 1\ 2)$ in table 4 and for $(\lambda\mu) = (\lambda_1 - 3\ 3)$ in table 5.

Table 4. $\langle(\lambda\ 0)L_1; (3\ 0)l\|(\lambda - 1\ 2)\kappa L\rangle$. $(\varphi(\lambda, L) = 2(\lambda+1)^2 - L(L+1).)$

	L_1	$\kappa = 0$	$\kappa = 2$
$\lambda - L =$ odd			
$l = 1$	$L - 1$	$\sqrt{\frac{(\lambda+L+2)L}{5(\lambda-1)(\lambda+1)(\lambda+2)(2L+1)\varphi(\lambda,L)}} \times (2\lambda(\lambda + L + 1) - (3L+2)(L-1))$	$-\frac{2}{5}\sqrt{\frac{5(\lambda-L+1)(L-1)(L+1)(L+2)}{(\lambda-1)(2L+1)\varphi(\lambda,L)}}$
	$L + 1$	$-\sqrt{\frac{(\lambda-L+1)(L+1)}{5(\lambda-1)(\lambda+1)(\lambda+2)(2L+1)\varphi(\lambda,L)}} \times (2\lambda(\lambda - L) - (3L+1)(L+2))$	$-\frac{2}{5}\sqrt{\frac{5(\lambda+L+2)(L+2)L(L-1)}{(\lambda-1)(2L+1)\varphi(\lambda,L)}}$
$l = 3$	$L - 1$	$\sqrt{\frac{(\lambda+L+2)(L-1)L(L+1)}{5(\lambda-1)(\lambda+1)(\lambda+2)(2L+1)(2L-3)(2L+3)\varphi(\lambda,L)}} \times (\lambda(3\lambda - 2L + 3) + 3L^2 - L - 12)$	$\sqrt{\frac{(\lambda-L+1)(L+2)}{5(\lambda-1)(2L-3)(2L+1)(2L+3)\varphi(\lambda,L)}} \times (7L^2 - 27 + 5\lambda L - 15\lambda)$
	$L + 1$	$-\sqrt{\frac{(\lambda-L+1)(L+2)(L+1)L}{5(\lambda-1)(\lambda+1)(\lambda+2)(2L-1)(2L+1)(2L+5)\varphi(\lambda,L)}} \times (\lambda(3\lambda + 2L + 5) + 3L^2 + 7L - 8)$	$\sqrt{\frac{(\lambda+L+2)(L-1)}{5(\lambda-1)(2L-1)(2L+1)(2L+5)\varphi(\lambda,L)}} \times (7L^2 + 14L - 5\lambda L - 20\lambda - 20)$
	$L - 3$	$-\sqrt{\frac{3(\lambda+L)(\lambda+L+2)(\lambda-L+3)(L-1)(L-2)L}{(\lambda-1)(\lambda+1)(\lambda+2)(2L-3)(2L-1)(2L+1)\varphi(\lambda,L)}} \times (\lambda - L)$	$\sqrt{\frac{3(\lambda+L)(\lambda-L+3)(\lambda-L+1)(L-2)(L+1)(L+2)}{(\lambda-1)(2L-3)(2L-1)(2L+1)\varphi(\lambda,L)}}$
	$L + 3$	$\sqrt{\frac{3(\lambda-L-1)(\lambda+L+4)(\lambda-L+1)(L+3)(L+2)(L+1)}{(\lambda-1)(\lambda+1)(\lambda+2)(2L+1)(2L+3)(2L+5)\varphi(\lambda,L)}} \times (\lambda + L + 1)$	$-\sqrt{\frac{3(\lambda-L-1)(\lambda+L+4)(\lambda+L+2)(L+3)L(L-1)}{(\lambda-1)(2L+3)(2L+5)(2L+1)\varphi(\lambda,L)}}$
$\lambda - L =$ even			
$l = 1$	L	$\sqrt{\frac{2(L-1)(L+2)}{5(\lambda-1)(\lambda+1)}}$	
$l = 3$	$L - 2$	$-\sqrt{\frac{2(\lambda-L+2)(\lambda+L+1)(L+2)(L-2)}{(\lambda-1)(\lambda+1)(2L-1)(2L+1)}}$	
	$L + 2$	$\sqrt{\frac{2(\lambda-L)(\lambda+L+3)(L-1)(L+3)}{(\lambda-1)(\lambda+1)(2L+1)(2L+3)}}$	
	L	$\sqrt{\frac{3}{5(\lambda-1)(\lambda+1)(2L-1)(2L+3)}}(5\lambda + 9 - 2L(L+1))$	

Table 5. $\langle(\lambda\ 0)L_1;(3\ 0)l\|(\lambda-3\ 3)\kappa L\rangle$. ($\chi(\lambda,L)=4(\lambda+1)^2-(L-1)(L+2)$; $W(\lambda,L)=4\lambda(\lambda+2)-3(L-1)(L+2)$.)

	L_1	$\kappa=1$	$\kappa=3$
$\lambda-L=$ odd			
$l=1$	$L-1$	$-\sqrt{\frac{(\lambda-L+1)(L+1)W(\lambda-2,L)}{5(\lambda-1)\lambda(\lambda+1)(2L+1)}}$	0
	$L+1$	$-\sqrt{\frac{(\lambda+L+2)LW(\lambda-2,L)}{5(\lambda-1)\lambda(\lambda+1)(2L+1)}}$	0
$l=3$	$L-1$	$-\sqrt{\frac{(\lambda-L+1)(L-1)}{5(\lambda-1)\lambda(\lambda+1)(2L-3)(2L+3)(2L+1)W(\lambda-2,L)}}$ $\times(\lambda(\lambda-2)(L+6)+3(L+2)(L^2-6))$	$-\sqrt{\frac{15(\lambda-L+1)(\lambda+L)(\lambda-L-1)(L-2)(L+2)(L+3)}{(\lambda-1)(2L-3)(2L+1)(2L+3)W(\lambda-2,L)}}$
	$L+1$	$-\sqrt{\frac{(\lambda+L+2)(L+2)}{5(\lambda-1)\lambda(\lambda+1)(2L-1)(2L+1)(2L+5)W(\lambda-2,L)}}$ $\times(\lambda(\lambda-2)(L-5)+3(L-1)(L^2+2L-5))$	$-\sqrt{\frac{15(\lambda-L-1)(\lambda+L+2)(\lambda+L)(L-2)(L-1)(L+3)}{(\lambda-1)(2L+1)(2L-1)(2L+5)W(\lambda-2,L)}}$
	$L-3$	$\sqrt{\frac{3(\lambda-L+3)(\lambda-L+1)(\lambda+L)(L-2)(L-1)(L+1)}{(\lambda-1)\lambda(\lambda+1)(2L-3)(2L-1)(2L+1)W(\lambda-2,L)}}$ $\times(\lambda-L-2)$	$-\sqrt{\frac{(\lambda-L+1)(\lambda-L-1)(\lambda-L+3)(L+1)(L+2)(L+3)}{(\lambda-1)(2L-3)(2L-1)(2L+1)W(\lambda-2,L)}}$
	$L+3$	$\sqrt{\frac{3(\lambda-L-1)(\lambda+L+4)(\lambda+L+2)(L+2)(L+3)L}{(\lambda-1)\lambda(\lambda+1)(2L+1)(2L+3)(2L+5)W(\lambda-2,L)}}$ $\times(\lambda+L-1)$	$-\sqrt{\frac{(\lambda+L+2)(\lambda+L+4)(\lambda+L)(L-2)(L-1)L}{(\lambda-1)(2L+1)(2L+3)(2L+5)W(\lambda-2,L)}}$
$\lambda-L=$ even			
$l=1$	L	$\sqrt{\frac{\chi(\lambda-2,L)}{5(\lambda-1)\lambda}}$	0
$l=3$	$L-2$	$\sqrt{\frac{(\lambda-L+2)(\lambda+L+1)(L-2)(L-1)}{(\lambda-1)\lambda(2L-1)(2L+1)\chi(\lambda-2,L)}}(-\lambda+L+3)$	$\sqrt{\frac{3(\lambda-L)(\lambda-L+2)(L+2)(L+3)}{(2L-1)(2L+1)\chi(\lambda-2,L)}}$
	$L+2$	$\sqrt{\frac{(\lambda-L)(\lambda+L+3)(L+2)(L+3)}{(\lambda-1)\lambda(2L+1)(2L+3)\chi(\lambda-2,L)}}(-\lambda-L+2)$	$\sqrt{\frac{3(\lambda+L+1)(\lambda+L+3)(L-1)(L-2)}{(2L+1)(2L+3)\chi(\lambda-2,L)}}$
	L	$\sqrt{\frac{6(L+2)(L-1)}{5(\lambda-1)\lambda(2L+3)(2L-1)\chi(\lambda-2,L)}}$ $\times(\lambda^2-2\lambda-6+L(L+1))$	$\sqrt{\frac{10(\lambda-L)(\lambda+L+1)(L+3)(L-2)}{(2L-1)(2L+3)\chi(\lambda-2,L)}}$

We have observed the following property for the Wigner coefficients: $\langle(\lambda_1\ 0)L-q;(3\ 0)l\|(\lambda\mu)\kappa L\rangle$ and $\langle(\lambda_1\ 0)L+q;(3\ 0)l\|(\lambda\mu)\kappa L\rangle$ can be obtained from one another by the substitution $L\to-(L+1)$ apart from an overall sign. A similar property is observed in the $R(5)\supset R(3)$ Wigner coefficient [20]. Another property is observed for the $SU(3)$ U-function. The U-function depends on the $SU(3)$ representations only, and does not depend on the $R(3)$ irreducible representation. Therefore the U-function can be obtained from the mathematical basis $SU(3)\supset SU(2)$. There is one property for the $SU(N)$ group in the Gelfand basis that its representation does not depend on the specific N [11, 21]. That is $\langle\Gamma_{1N}\Gamma_{1N-1};\Gamma_{2N}\Gamma_{2N-1}\|\Gamma_{3N}\Gamma_{3N-1}\rangle=\langle\Gamma_{1N'}\Gamma_{1N'-1};\Gamma_{2N'}\Gamma_{2N'-1}\|\Gamma_{3N'}\Gamma_{3N'-1}\rangle$, for arbitrary N and N' as long as all the irreducible representations are allowed. Here Γ_N is the irreducible representation of $SU(N)$ and Γ_{N-1} is the irreducible representation of $SU(N-1)$. In our case, the $SU(3)$ irreducible representations are also allowed in the $SU(2)$ group. We can treat the $SU(3)$ representations as $O(3)\approx SU(2)$ representations. They are related by $(\lambda\mu)\to L=\lambda/2$. Therefore the $SU(3)$ U-function can be written explicitly in terms of the ordinary 6-j symbol, or Racah coefficient, in the following:

$$U((\lambda_1\ 0)(2\ 0)(\lambda\mu)(1\ 0);(\lambda_{12}\mu_{12})(3\ 0)) = U\left(\frac{\lambda_1}{2}1\frac{\lambda}{2}\frac{1}{2};\frac{\lambda_{12}}{2}\frac{3}{2}\right)$$
$$= (-1)^{(\lambda_1/2+1+\lambda/2+1/2)}\sqrt{(2(\lambda_{12}/2)+1)(2(3/2)+1)}\left\{\begin{array}{ccc}\lambda_1/2 & 1 & \lambda_{12}\\ 1/2 & \lambda/2 & 3/2\end{array}\right\}. \tag{3}$$

It is easily checked that the U-function in table 1 satisfies (3).

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References

[1] Elliott J P 1958 *Proc. R. Soc.* A **245** 128, 562
[2] Elliott J P and Harvey M 1963 *Proc. R. Soc.* A **272** 557
[3] Hecht K T 1965 *Nucl. Phys.* **62** 1
[4] Engeland T 1965 *Nucl. Phys.* **72** 67
[5] Banerjee M K and Levinson C A 1963 *Phys. Rev.* **130** 1036, 1064
[6] Koltun D 1961 *Phys. Rev.* **124** 1162
[7] Brink D M and Nash G F 1963 *Nucl. Phys.* **40** 608
[8] Flores J and Monshinsky M 1967 *Nucl. Phys.* A **93** 81 and references therein
[9] Gell-mann M and Ne'eman Y 1964 *The Eightfold Way* (New York: Benjamin)
[10] DeSwart J J 1962 *Rev. Mod. Phys.* **34** 916
[11] Sun H Z 1980 *High Energy Nucl. Phys.* 4 73
[12] Hecht K T 1990 *J. Math. Phys.* **31** 2781
[13] Arima A and Iachello F 1975 *Phys. Rev. Lett.* **35** 1069
[14] Vergados J D 1968 *Nucl. Phys.* A **111** 681
[15] Draayer J P and Akiyama Y 1973 *J. Math. Phys.* **14** 1904
[16] Akiyama Y and Draayer J P 1973 *Comput. Phys. Commun.* **5** 405
[17] For a review see Hecht K T 1987 *The Vector Coherent State Method and its Application to Problems of Higher Symmetries* (Lecture Notes in Physics 290) (Berlin: Springer)
[18] Hecht K T 1990 *J. Phys. A: Math. Gen.* **23** 407
[19] For example, see Liu Y X, Sun H Z and Zhao E G 1994 *J. Phys. G: Nucl. Phys.* **20** 407
[20] Elliott J P, Evans J A and Long G L 1992 *J. Phys. A: Math. Gen.* **25** 4622
[21] Chen J Q *et al* 1979 *High Energy Nucl. Phys.* 3 216